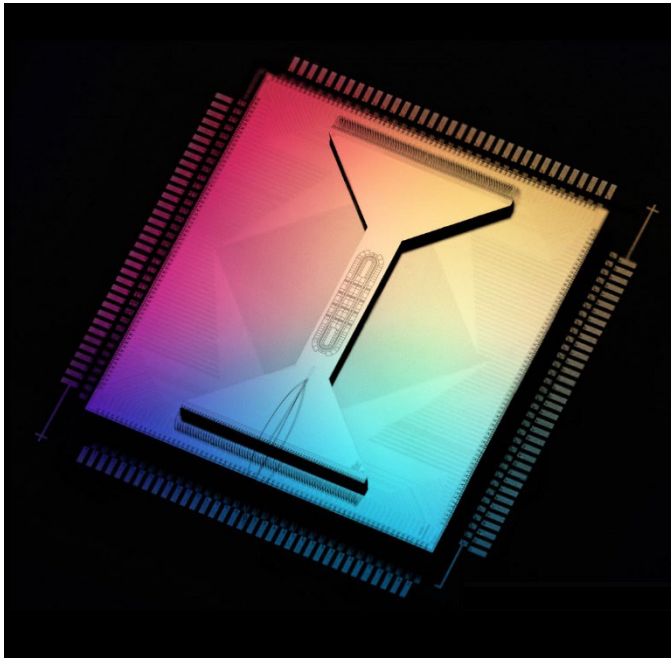


PHYS-541 - Quantum Computing

Vincenzo Savona

EPFL Center for Quantum Science and Engineering



Trapped-ion quantum processor: Quantinuum



Superconducting quantum computer: IBM

PHYS-541 - Quantum Computing

Teacher: *Vincenzo Savona*

Assistants: *Clemens Giuliani, Sara Alves, Khurshed Fitter*

Dates: Thursdays 12.9 – 19.12.2024

Time: Course 13:15 – 16:00

Exercises 16:15 – 18:00

Location: BS 260

Material: see moodle (PHYS-541)

Sources: M. A. Nielsen & I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge, 2011)

John Preskill, *Lecture Notes on Quantum Information and Computation*

http://theory.caltech.edu/~preskill/ph219/ph219_2019-20

In October 2019, Google announced **quantum supremacy**:

A programmable, general-purpose engineered quantum device could perform a computational task much faster than any existing supercomputer would do

Nature **574**, 505 (2019)

Scope of the course:

Acquire the skills needed to understand this result

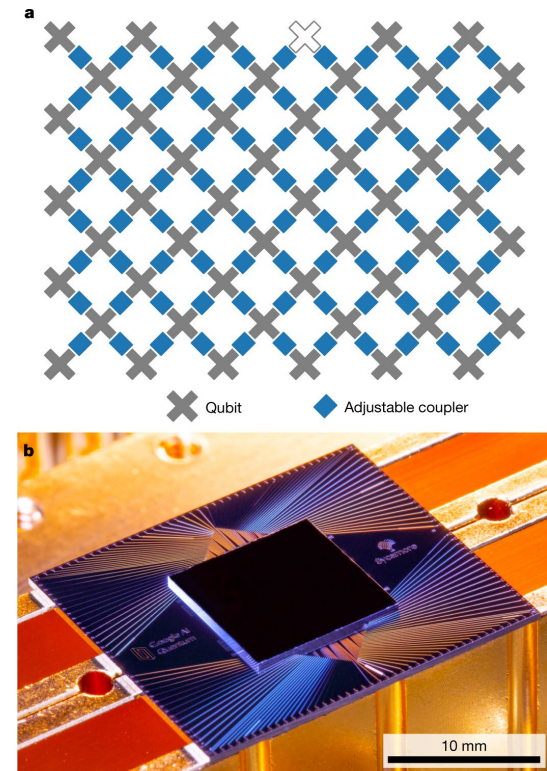
Notion of quantum information

Paradigm of digital quantum computing

Notion of classical and quantum computational complexity

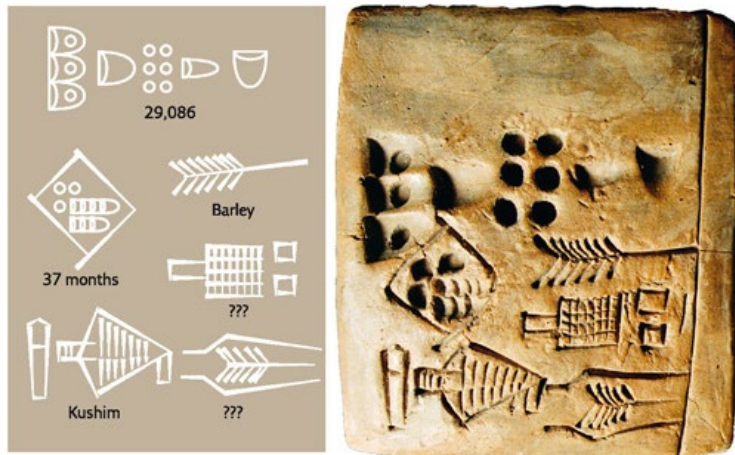
Notion of errors in a quantum computation

Quantum circuits and algorithms



The physical nature of information

Information is physical



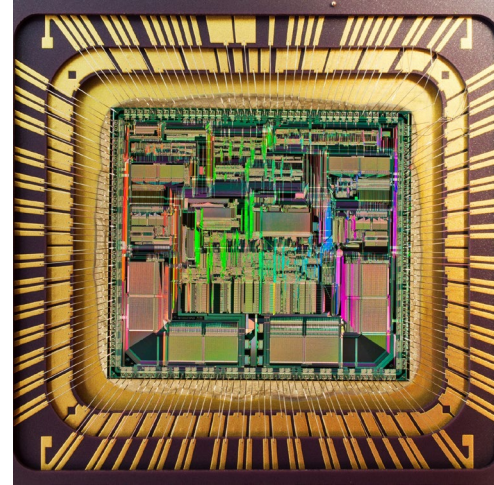
“Kushim” clay tablet (3400 – 3000 BC)



“Quipu” (3000 – 2000 BC)

The physical nature of information

Information is physical



Current information devices are described by the **laws of classical physics**

Everyday phenomena obey the **laws of non-relativistic quantum mechanics**

Quantum superposition and **entanglement** are not used in classical devices

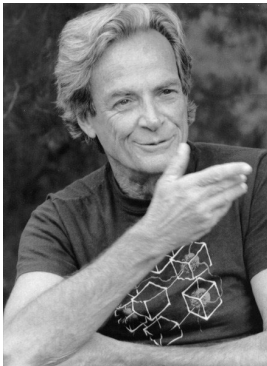
Can these properties result in a more efficient computation paradigm?

The idea of quantum computing

Time-dependent Schrödinger equation:

$$|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$$

Quantum many-body systems are in general computationally untreatable: resources scale exponentially with the size of the system



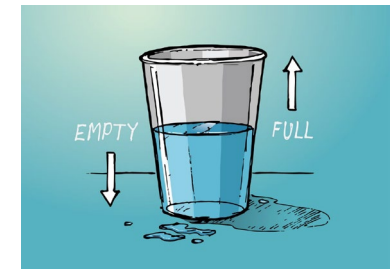
“... nature isn’t classical, dammit, and if you want to make a simulation of nature, you’d better make it quantum mechanical, and by golly it’s a wonderful problem, because it doesn’t look so easy.”

Richard Feynman
Simulating physics with computers (1981)

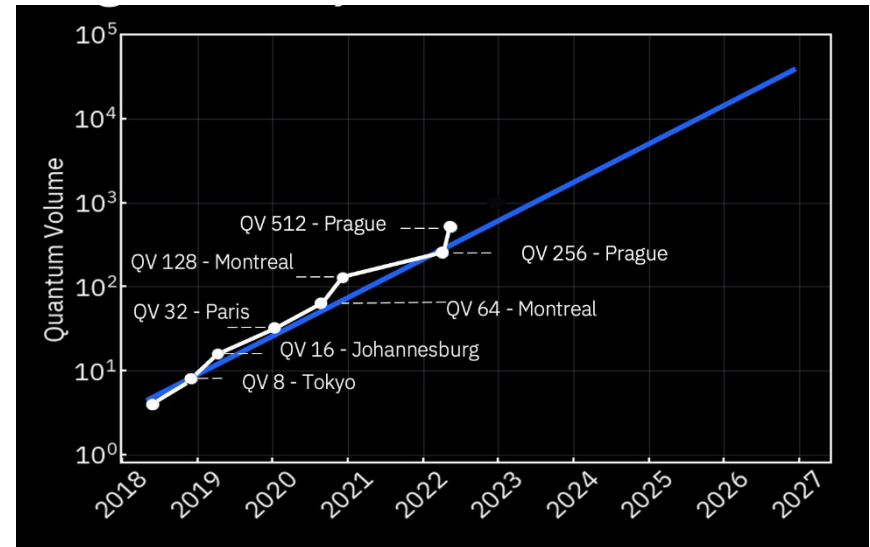
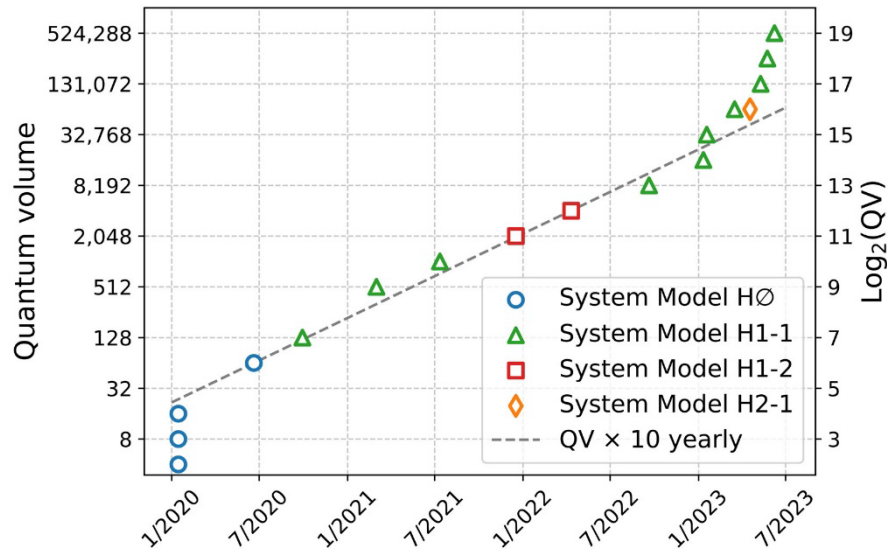
A resource, not a limitation!

Nature executes this specific “computational” task exponentially faster than classical computers

If we could map a computational task onto $|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$ efficiently, we would have a great computational advantage

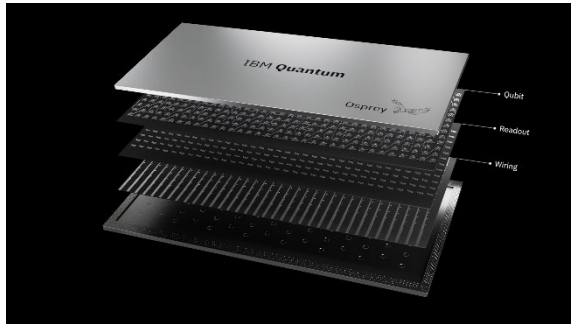


Advances in quantum computing

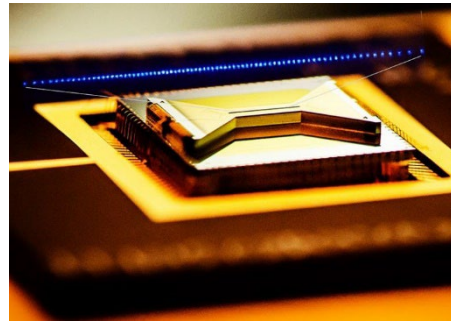


Many candidate quantum computing platforms

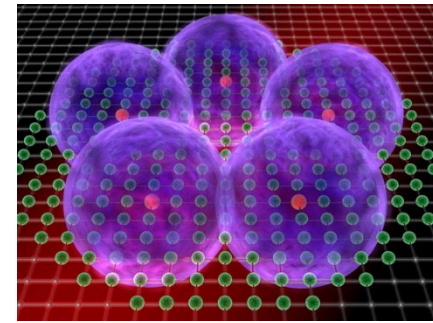
Superconducting circuits



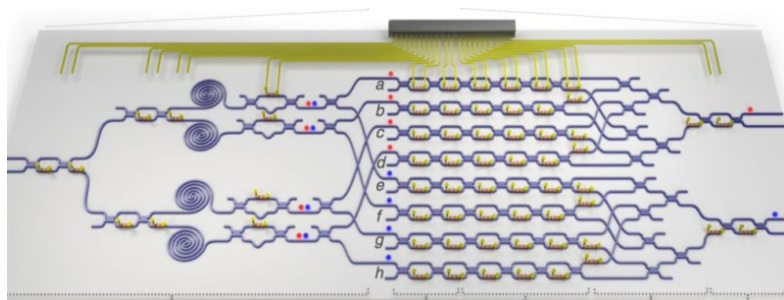
Trapped ions



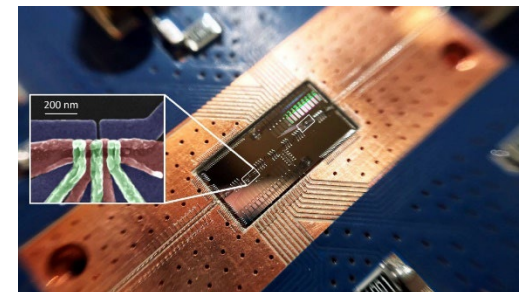
Rydberg atoms



Integrated photonics

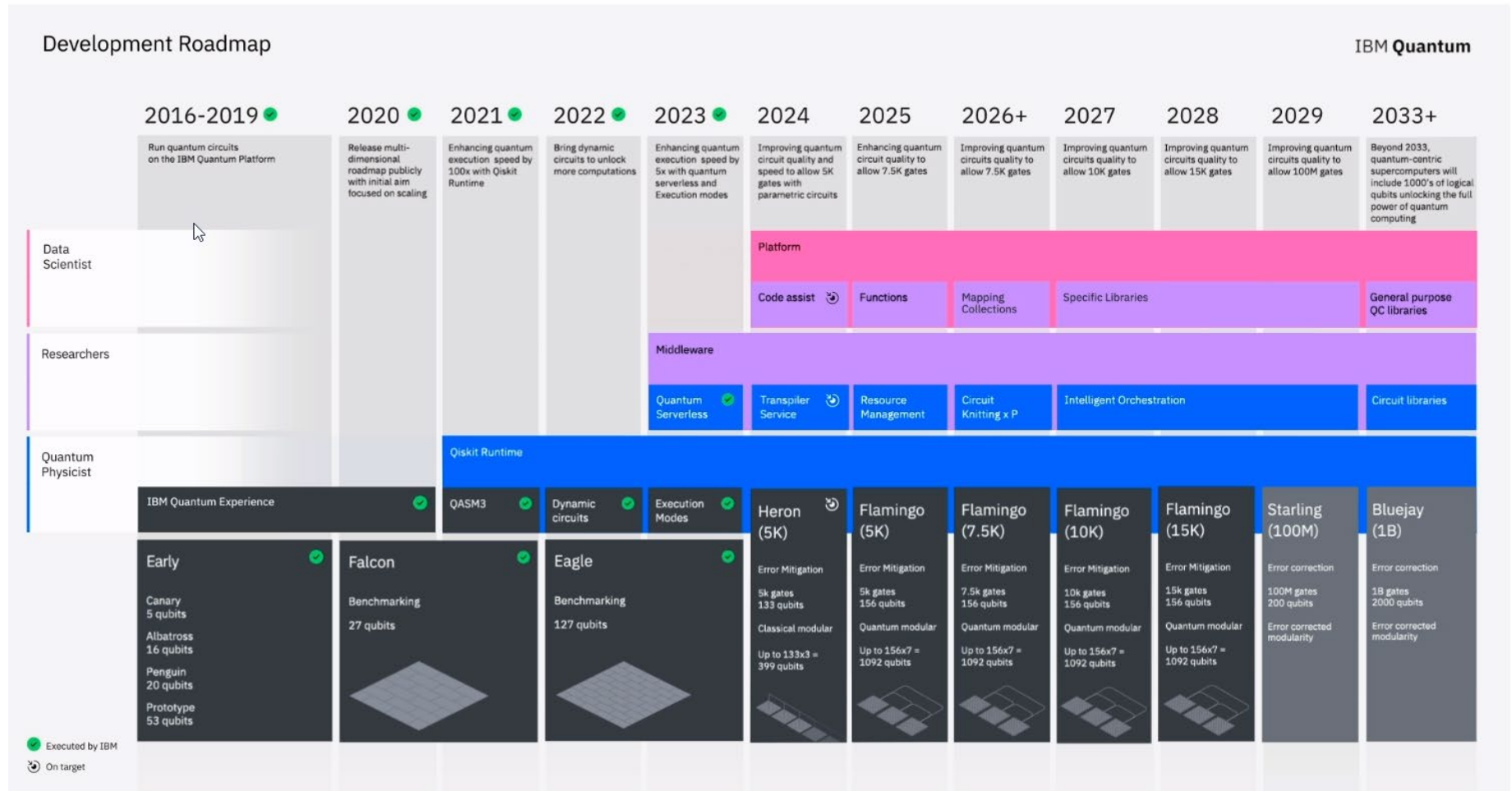


Electron (or hole) spin



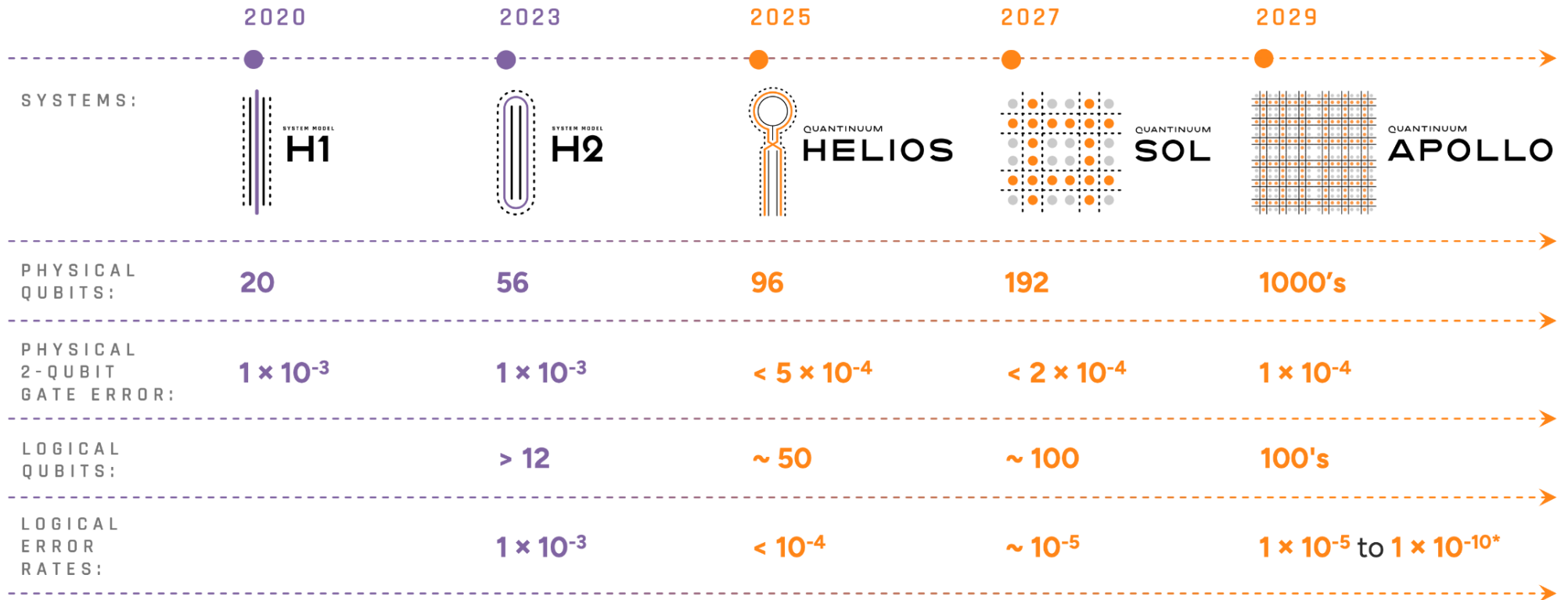
Advances in quantum computing

IBM Quantum Computing Roadmap 2024



Advances in quantum computing

Development roadmap

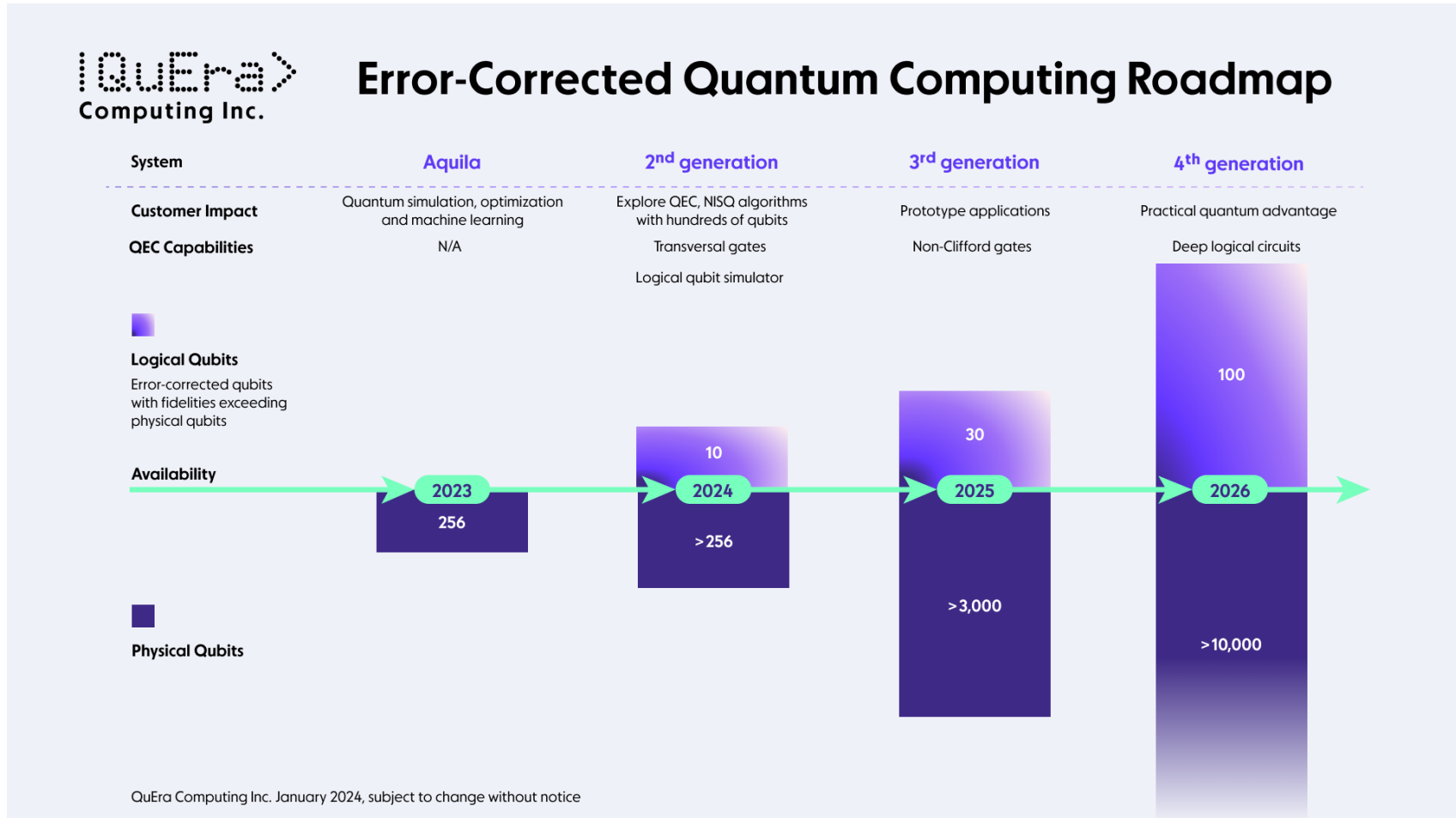


© 2024 Quantinuum. All Rights Reserved.

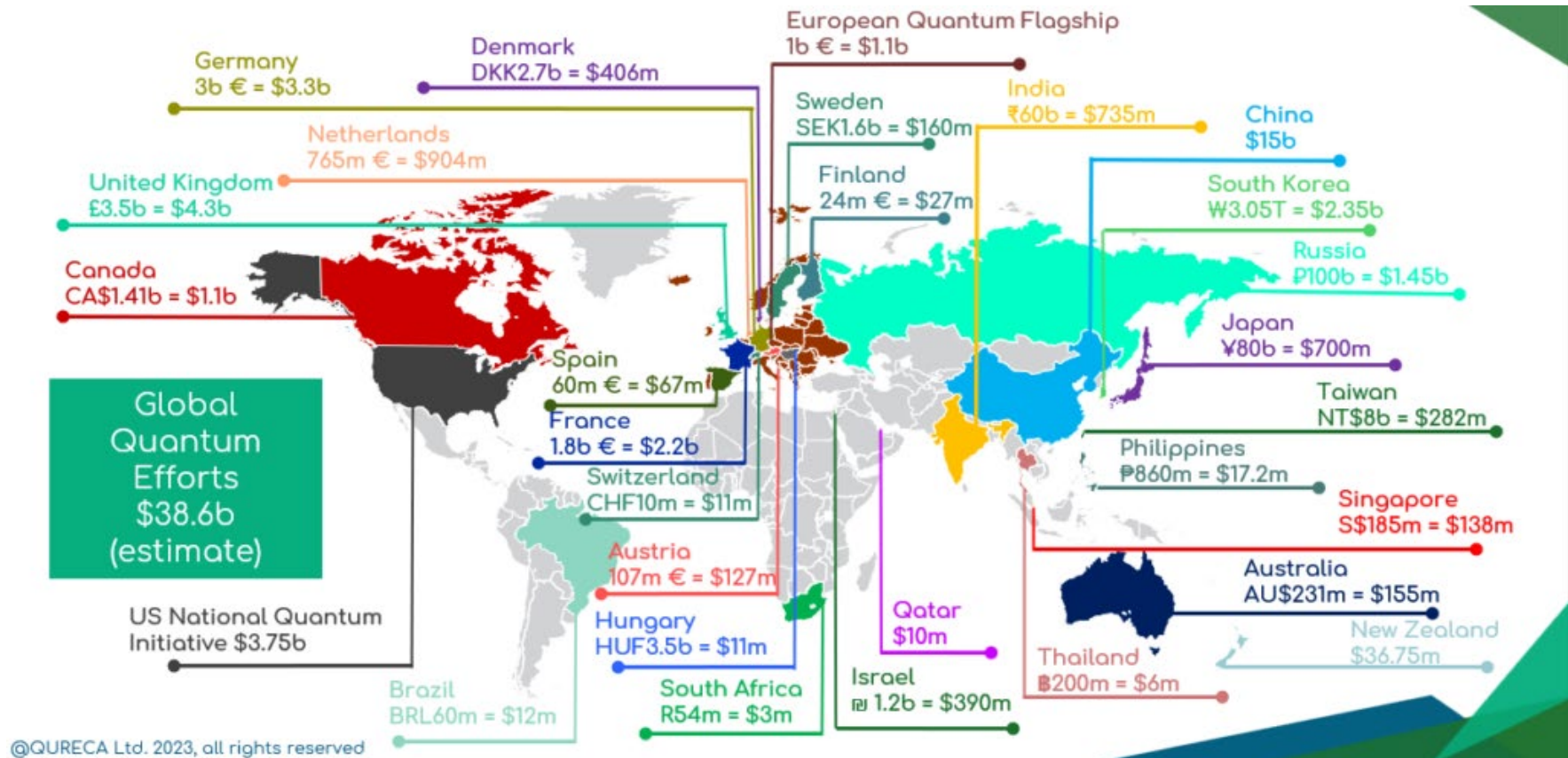
*analysis based on recent literature in new, novel error correcting codes predict that error could be as low as $1E-10$ in Apollo (ref: [arXiv:2403.16054](#), [arXiv:2308.07915](#))

Advances in quantum computing

QuEra neutral-atom (Rydberg) quantum computer



Public investment in quantum computing



A growing ecosystem

Quantum computer makers

Superconducting circuits



Silicon, carbon, & helium



Developer & programming tools



Quantum hardware components



Photonics



Neutral atoms



Trapped ions



Qubit control & error correction



Enterprise use cases

Cross-industry applications



Drug discovery



Financial services



Chemical & materials simulation



Optimization & logistics



What can a quantum computer do?

A widespread concept is that of “quantum parallelism”

$$|\psi\rangle = \left| \text{cat}_1 \right\rangle + \left| \text{cat}_2 \right\rangle + \dots + \left| \text{cat}_n \right\rangle$$

A single quantum register can apparently store **exponentially more information** than a classical one

One operation on the quantum register will be carried out “**in parallel**” on all stored items

This simple picture of quantum parallelism is not useful. A **readout** of the quantum register is a **quantum measurement**. It will **return a random item** and destroy the remaining information through **state collapse**

$$|\psi\rangle = \left| \text{cat}_1 \right\rangle + \left| \text{cat}_2 \right\rangle + \dots + \left| \text{cat}_n \right\rangle$$

↓ readout

$$|\psi\rangle = \left| \text{cat}_2 \right\rangle$$

Need for algorithms that use quantum superposition to take advantage of quantum parallelism

What can a quantum computer do?

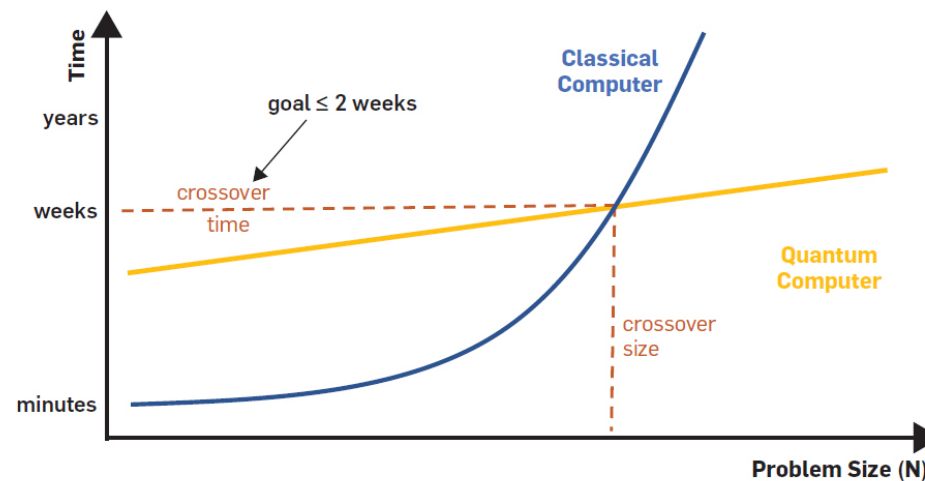
Quantum computers can solve **some** computational problems **better** than conventional computers

some: A quantum computer is not a universal tool

Among problems with **quantum advantage**, many have very high societal benefit

better: **Quantum advantage** is not just “faster”

It is about how **computational time** scales with **problem size**



A quantum algorithm zoo

<https://quantumalgorithmzoo.org/>

Today there are **hundreds of useful quantum algorithms** ready for future quantum hardware

Quantum Phase estimation (1995).

Estimate an eigenvalue of a unitary operator with error ε , using $O(\log(1/\varepsilon))$ qubits and $O(1/\varepsilon)$ operations. Used as a primitive in many algorithms, like Shor or HHL. **Simulate energy levels of complex Hamiltonians efficiently.**

Quantum Amplitude estimation / amplification (2000).

Estimate or amplify one component in a given quantum state. Useful primitive in several algorithms, like e.g. Grover's algorithm or Quantum accelerated Monte Carlo sampling.

Quantum Fourier Transform (1994).

Compute discrete Fourier transform of 2^n amplitudes with complexity $O(n^2)$. Primitive ubiquitous in many computational tasks, from Shor to data science.

A quantum algorithm zoo

<https://quantumalgorithmzoo.org/>

Today there are **hundreds of useful quantum algorithms** ready for future quantum hardware

Shor's algorithm (1994).

Compute a prime factor of a n -qubit integer with $O(n^2 \log(n) \log(\log(n)))$ gates. Best known classical algorithm requires $\exp(O(n^{1/3} \log(n)^{2/3}))$ time

Grover's algorithm (1996).

Search an unstructured database of N entries with $O(N^{1/2})$ gates. Best classical algorithm requires time $O(N)$. There's proof that $O(N^{1/2})$ is optimal according to quantum mechanics. Evidence that quantum computers can't solve NP-complete problems.

Digital quantum simulation (1996).

Compute $U = e^{-iHt}$ on n qubits with $O(n^3 \log(n))$ gates. Classical algorithms are exponential.

A quantum algorithm zoo

<https://quantumalgorithmzoo.org/>

Today there are **hundreds of useful quantum algorithms** ready for future quantum hardware

Quantum solution of linear systems of equations (HHL algorithm) (2008).

Estimate (a measurement on) **the solution of a linear system of N equations** with $O(\log(N))$ complexity. Ubiquitous applications: electromagnetic scattering, linear differential equations, finite element simulations, least-square fitting, machine learning and data science.

Quantum-accelerated Monte Carlo sampling (2015).

Sample a function of a random variable (st.d. σ) with accuracy ε , using $O(\sigma/\varepsilon)$ samples, instead of $O(\sigma^2/\varepsilon^2)$. Ubiquitous use in science, and finance (risk analysis, derivative pricing)

How to deal with errors

Digital electronics is subject to errors (cosmic rays!). **Today's error rate is 10^{-10} errors/bit/hour**

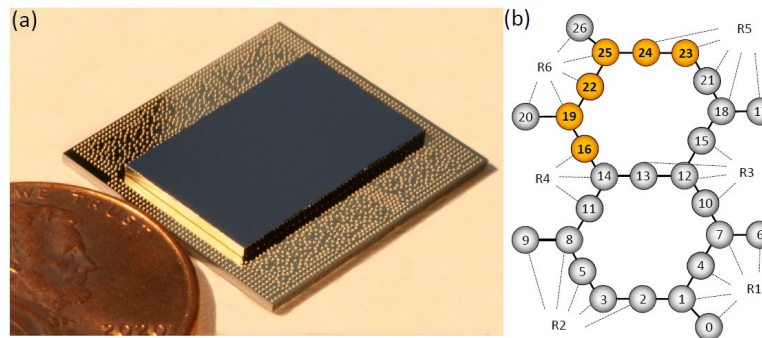
https://en.wikipedia.org/wiki/ECC_memory

Errors in digital electronics are not corrected!

<https://arstechnica.com/gadgets/2021/01/linus-torvalds-blames-intel-for-lack-of-ecc-ram-in-consumer-pcs/>

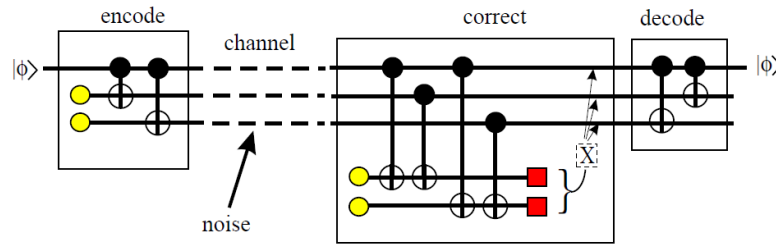
In quantum computers errors are an issue for two reasons

1. Error rates. One-qubit gate: 3.8×10^{-4} ; two-qubit gate: 6.4×10^{-3} ; one-qubit readout: 6.0×10^{-3}
IBM 27-qubit Quantum Falcon Processor, [arXiv:2008.08571](https://arxiv.org/abs/2008.08571)
2. Error correction requires information readout, which is a destructive process because of collapse. Need an agnostic error correction scheme



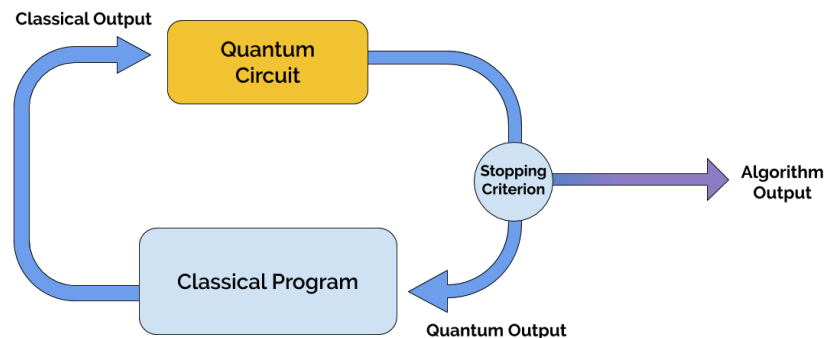
How to deal with errors: two roads

Fault-tolerant quantum computing: Correct errors with Quantum Error Correction Codes (QECC)



Timescale to achievement: decades

Hybrid algorithms on Noisy Intermediate-Scale Quantum (NISQ) hardware: Do not correct errors. Make quantum subroutines as short (“shallow”) as possible. Combine with classical processing. Estimate result from statistical inference of noisy output. Use error mitigation schemes.



Timescale to achievement: years

Noisy intermediate-scale quantum algorithms, Rev. Mod. Phys. 94, 015004 (2022)

Variational quantum algorithms, Nature Reviews Physics 3, 625 (2021)

Hybrid variational quantum algorithms

Among the most important Variational Quantum Algorithms are:

[The Variational Quantum Eigensolver \(2014\).](#)

Estimate the ground state energy of a quantum system using a parametrized representation of the quantum state. Holds great promise for the simulation of molecules and materials.

[The Quantum Approximate Optimization Algorithms \(2014\).](#)

Finds an approximate solution to a discrete unconstrained optimization problem. It is a digital version of the quantum annealing process. Many applications in industrial processes, transportation, climate, medicine, etc.

[The Quantum Variational Dynamics \(2017\).](#)

Estimate the time evolution of a quantum state governed by a given Hamiltonian, using a parametrized representation of the state.

Outline

1. General introduction
2. A short overview of quantum mechanics
3. The paradigm of digital quantum computing
4. Universal quantum gates and the Solovay-Kitaev theorem
5. Deutsch and Deutsch-Jozsa algorithms
6. Shor's factoring algorithm
7. Grover's search algorithm
8. Overview of other algorithms
9. The theory of open quantum systems and noisy quantum channels
10. Errors and quantum error correction
11. Fault-tolerant quantum error correction
12. Hybrid quantum algorithms: the variational quantum eigensolver
13. Hybrid quantum algorithms: the quantum approximate optimization algorithm
14. Hybrid quantum algorithms: the variational quantum dynamics simulation
15. Current challenges in quantum computing: an outlook.

